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A genetic algorithm for inverse radiation problems

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Abstract—An inverse radiation analysis for simultaneous estimation of the single scattering albedo, the optical thickness and the phase function, from the knowledge of the exit radiation intensities is presented. A genetic algorithm is adopted as the optimizer to search the parameters of the radiation system. The study shows that the single scattering albedo and the optical thickness can be estimated accurately even with noisy data. The estimation of the phase function is more difficult than that of the single scattering albedo and the optical thickness. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

Radiation heat transfer in a participating medium has numerous engineering applications in a variety of areas such as fibrous insulation, glass manufacture and energy conservation. Although several techniques are available for the solutions of the radiation heat transfer problems, the radiative properties of the medium, such as the single scattering albedo, the extinction coefficient and the phase function, are uncertain. Inverse radiation analysis is concerned with the determination of the radiative properties from various types of radiation measurements. Many studies have been reported for the solution of the inverse problems [1–7]. A general inverse radiation problem consists of determining the single scattering albedo, the optical thickness and the phase function. The problem is difficult to solve because the phase function is represented by a series of the Legendre polynomials; therefore a large number of expansion coefficients must be estimated. Step function [4] and Henyey–Greenstein function [5] approximations for the phase function have been proposed to reduce the unknowns to be evaluated. Kamiuto [6] has estimated a complete profile of the phase function and the single scattering albedo. In the paper, the optical thickness has considered to be known or determined by the light-transmission method. An extensive review of the inverse problems has been given in a series of papers by McCormic [8–10].

Genetic algorithms [11] are the search methods that combine the concept of survival-of-the-fittest among string patterns with a regulated yet randomized information exchange. In 1962, Holland [12] explored the adaptive process in artificial systems. Based on Fraser's search algorithm [13, 14] in natural systems, the Holland adaptive process is an artificial algorithm in which the system can adjust itself to changes in the environment. Five years later, Bagley [15] named this adaptive process a genetic algorithm and constructed a genetic algorithm to search parameter sets in a game-playing problem. Later on, Hollstien [16] applied genetic algorithms to a digital feedback problem in a computer control system. With the progress in genetic algorithm theory, many applications have been employed. These applications included mass-spring-dashpot system identification, gas pipeline design, VLSI circuit layout, etc. [17]. In those researches, the genetic algorithms appear both global and efficient over a broad spectrum of problems; two major features contributed to this belief. Firstly, genetic algorithms have a distinctive pattern in a search sample. Instead of a point-to-point search, the search sample is expanded to a group in which a search point moves among peaks. Consequently, the probability of finding a near-global optimal solution is raised. Another feature is the use of payoff (objective function) information instead of using derivatives (sensitivity analysis) or other auxiliary knowledge. Thus, they are not restricted to the conditions of continuity, sensitivity, convexity, monotonicity and nonlinearity of both objective functions and constraints. In this paper, we apply a genetic algorithm to solve the inverse problem for simultaneously determining the single scattering

NOMENCLATURE

a_n	expansion coefficients for the phase function
$\mathbf{b}$	$[\omega, \tau_0, a_0, a_1, \ldots, a_N]^T$
g_n	Chandrasekhar polynomials
I	radiation intensity
J	objective function
P_n	Legendre polynomials
p	phase function
Y	measured exit radiation intensities at the surface $\tau = 0$
Z	measured exit radiation intensities at the surface $\tau = \tau_0$.

Greek symbols

μ	direction cosine
σ	standard deviation
τ	optical variable
τ_0	optical thickness
ω	single scattering albedo
ξ	eigenvalues
ζ	random variable.

Superscripts

T	transpose.

albedo, the optical thickness and the phase function, from the knowledge of the exit radiation intensities.

ANALYSIS

Consider an azimuthally symmetric, absorbing, anisotropically scattering, gray, plane-parallel medium, of optical thickness τ_0, subjected to isotropic incident radiation at the boundary surface $\tau = 0$ and no external incident radiation at $\tau = \tau_0$. The equation of radiative transfer and the boundary conditions can be written as [18]

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu') I(\tau,\mu')\,d\mu' \quad (1)$$

$$I(0,\mu) = 1 \quad \mu > 0 \quad (2)$$

$$I(\tau_0, -\mu) = 0 \quad \mu > 0. \quad (3)$$

Here, $I(\tau, \mu)$ is the radiation intensity, τ is the optical variable, μ is the cosine of the angle between the direction of the radiation intensity and the positive τ axis, ω is the single scattering albedo, and $p(\mu, \mu')$ is the phase function, which is expressed in terms of the Legendre polynomials as

$$p(\mu,\mu') = \sum_{n=0}^{N^*} a_n P_n(\mu) P_n(\mu'). \quad (4)$$

Using the P_N method [19], the solution for the radiation intensity $I(\tau, \mu)$ is expressed in the form

$$I(\tau,\mu) = \sum_{n=0}^{N} \frac{2n+1}{2} P_n(\mu) \times \sum_{j=1}^{J^*} \left[A_j e^{-\tau/\xi_j} + (-1)^n B_j e^{-(\tau_0-\tau)/\xi_j} \right] g_n(\xi_j) \quad (5)$$

where $J^* = (N+1)/2$, N is an odd integer, $g_n(\xi)$ are the Chandrasekhar polynomials determined from the recurrence formula

$$(n+1) g_{n+1}(\xi) = h_n \xi g_n(\xi) - n g_{n-1}(\xi) \quad (6)$$

for $n = 0,1, \ldots, N$, with $g_0(\xi) = 1$ and $h_n = 2n+1-\omega a_n$. The eigenvalues ξ_j, $j = 1,2, \ldots, J^*$, are the J^* positive solutions of the following eigenvalue problem:

$$\frac{n(n-1)}{h_n h_{n-1}} g_{n-2}(\xi) + \frac{1}{h_n}\left[\frac{(n+1)^2}{h_{n+1}} + \frac{n^2}{h_{n-1}}\right] \times g_n(\xi) + \frac{(n+2)(n+1)}{h_{n+1} h_n} g_{n+2}(\xi) = \xi^2 g_n(\xi) \quad (7)$$

for $n = 0,2,4, \ldots, N-1$.

The constants A_j and B_j are determined by requiring the solution given by equation (5) to satisfy the Marshak boundary condition [18], from which we can show that

$$\sum_{n=0}^{N} \frac{2n+1}{2} S_{\alpha,n} \sum_{j=1}^{J^*} \left[A_j + (-1)^n B_j e^{-\tau_0/\xi_j} \right] g_n(\xi_j) = S_{\alpha,0} \quad (8)$$

and

$$\sum_{n=0}^{N} \frac{2n+1}{2} S_{\alpha,n} \sum_{j=1}^{J^*} \left[(-1)^n A_j e^{-\tau_0/\xi_j} + B_j \right] g_n(\xi_j) = 0 \quad (9)$$

for $\alpha = 0,1, \ldots, (N-1)/2$, where

$$S_{\alpha,n} = \int_0^1 P_{2\alpha+1}(\mu) P_n(\mu)\,d\mu. \quad (10)$$

Once A_j and B_j are available, the exit radiation intensities at $\tau = 0$ and $\tau = \tau_0$ are calculated from [19]

$$I(0,-\mu) = \frac{\omega}{2} \sum_{n=0}^{N} a_n P_n(\mu) \times \sum_{j=1}^{J^*} \xi_j \left[(-1)^n A_j \frac{1 - e^{-\tau_0/\mu} e^{-\tau_0/\xi_j}}{\mu + \xi_j} + B_j \frac{e^{-\tau_0/\mu} - e^{-\tau_0/\xi_j}}{\mu - \xi_j} \right] \times g_n(\xi_j) \quad \mu > 0 \quad (11)$$

and

$$I(\tau_0,\mu) = e^{-\tau_0/\mu} + \frac{\omega}{2}\sum_{n=0}^{N} a_n P_n(\mu)$$

$$\times \sum_{j=1}^{J^*} \xi_j \left[A_j \frac{e^{-\tau_0/\mu} - e^{-\tau_0/\xi_j}}{\mu - \xi_j} + (-1)^n B_j \frac{1 - e^{-\tau_0/\mu} e^{-\tau_0/\xi_j}}{\mu + \xi_j} \right]$$

$$\times g_n(\xi_j) \quad \mu > 0. \quad (12)$$

The inverse radiation problem involves the determination of the single scattering albedo, the optical thickness, and the phase function from the knowledge of the exit radiation intensities. In order to solve this problem, we define the objective function

$$J(\mathbf{b}) = \int_{-1}^{0} [I(0,\mu;\mathbf{b}) - Y(\mu)]^2 \, d\mu + \int_{0}^{1} [I(\tau_0,\mu;\mathbf{b}) - Z(\mu)]^2 \, d\mu \quad (13)$$

where $Y(\mu)$ and $Z(\mu)$ are the measured exit radiation intensities at the surfaces $\tau = 0$ and $\tau = \tau_0$, respectively; $I(0, \mu; \mathbf{b})$ and $I(\tau_0, \mu; \mathbf{b})$ are the estimated exit radiation intensities at the surfaces $\tau = 0$ and $\tau = \tau_0$, respectively, by using an estimated vector $\mathbf{b} = [\omega, \tau_0, a_0, a_1, \ldots, a_N]^T$. The solution of the inverse problem is obtained by minimizing the value of the objective function with respect to the unknown parameters. A genetic algorithm is used for this optimization process, which is described next.

Genetic algorithms (GA) work iteration by iteration, successively generating and testing a population of strings. The process is similar to a natural population of biological creatures where successive generations of creatures are conceived, born and raised until they themselves are ready to reproduce. For illustration, a simple GA adopted from Goldberg's research [17] will be used. There are three operators in a simple GA: reproduction, crossover and mutation. After the initial population and its fitness (the value of the objective function) are give, the reproduction process starts. It duplicates part of the previous generation that has the preferable fitness. Then a crossover operation follows. It randomly mates pairs from the newly reproduced string, reassembles part of the parents strings and generates pairs of offspring to replace the parents. Finally, the mutation operator randomly changes the genes in a string through an assigned probability. Once the first trial is completed, the iterations will not stop until a satisfactory solution is reached.

Reproduction is a process in which individual strings are duplicated according to their objective function values. The values can be thought of as some measure of the maximized profit, utility, or goodness, such as minimum weight. In applying these values to weight optimization, it can be considered as the minimized weight. In that case, 'copying strings according to their fitness values' means that strings with a lower value have a higher probability of contributing one or more offspring in the next generation. The reproduction operator may be implemented in algorithmic form in a number of ways. The easiest is to create a biased roulette wheel, where each current string in the population has a roulette wheel slot sized according to its fitness. Each time a new offspring is required, a simple spin of the weighted roulette wheel yields the reproduction candidate. In this way more fit strings have a higher number of offspring in the succeeding generation. Once a string has been selected for reproduction, an exact replica of the string is made. This string is then entered into a mating pool, an intermediate new population, for further genetic operator action.

After reproduction, a simple crossover may proceed in two steps. Firstly, it randomly pairs members of the newly reproduced strings in the mating pool with uniform distribution. Secondly, genes in each pair of strings are interchanged as follows: a position, k, along the strings is selected uniformly at random between 1 and the string length, l, less one. Two new strings are created by swapping all characters between positions $k+1$ and l inclusively. Mutation plays a decidedly secondary role in the operation of tradition genetic algorithms. A mutation operator is needed because it prevents the occasional loss of potentially useful genetic material from reproduction and crossover. Even though the reproduction and crossover effectively search and reformulate the extant genes, occasionally they may become overzealous and drop some material. The function of the mutation operator is to protect against such an irrecoverable loss and is the occasional random alteration of the value of a string position.

RESULTS AND DISCUSSION

We now present numerical results to demonstrate the use of a genetic algorithm for simultaneously estimating the single scattering albedo, the optical thickness and the phase function, from the knowledge of the exit radiation intensities. In order to simulate the measured exit intensities with measurement errors, Y, and Z, random errors of standard deviation σ are added to the exact intensities computed from the solution of the direct problem. Thus, we have

$$Y_{\text{measured}} = Y_{\text{exact}} + \sigma\zeta \quad (14)$$

and

$$Z_{\text{measured}} = Z_{\text{exact}} + \sigma\zeta \quad (15)$$

where ζ is a random variable with normal distribution, zero mean and unit standard deviation. For all the results presented in this work, the exit radiation intensities are measured at the surfaces $\tau = 0$ and $\tau = \tau_0$, and 20 measurement points are taken at each surface over the polar angle interval $0 \leqslant \theta \leqslant \pi/2$.

A genetic algorithm, Genitor [20], is used to fulfil

Table 1. The coefficients of the phase function used in the calculation [21].

	Phase function I	Phase function II
n	a_n	a_n
0	1	1
1	2.00735	0.84664
2	2.74550	0.03635
3	2.79268	−0.04477
4	3.04641	0.33367
5	2.97857	0.13727
6	3.05581	0.02852
7	2.98020	0.00353
8	2.53995	0.00027
9	1.88679	
10	0.85129	
11	0.24523	
12	−0.02434	
13	0.12820	
14	0.10524	
15	−0.00671	
16	0.03863	
17	0.00100	
18	0.00036	
19	0.00002	

the examples. Genitor reproduces only one offspring which is recombined between two parents at each generation; each offspring is immediately evaluated and replaces the worst fitness. A rank-based selection scheme is used with a linear selective bias of two. The reduced surrogate strategy is used as the crossover operator. The adaptive mutation rate is set to 0.005. The string length is 20 bits for each variable. The population size is 5000 for example one and 2000 for example two, respectively.

In the first case, the single scattering albedo and the optical thickness are assumed to be 0.9 and 1, respectively. Phase function I of Table 1 [21] is used for the scattering characteristics of the medium, which is determined from the Mie theory [18]. The results of the inverse analysis are shown in Figs. 1 and 2. Figure 1 shows the best generation results in three different runs for simulated experimental data containing errors of standard deviation $\sigma = 0.01$. The trend of improvement is significant in the beginning generations in all three runs. After the early generations, the best results of the three runs are close to one another. With experimental errors $\sigma = 0.02$, the trend of improvement in three different runs is the same as that for $\sigma = 0.01$. Figure 2 shows the results obtained with the inverse analysis for $\sigma = 0.01$ and $\sigma = 0.02$. As may be seen from this figure, the estimated values are close to their exact values. Increasing σ from 0.01 to 0.02, the accuracy of the estimation decreases.

Figures 3 and 4 show the results of the inverse

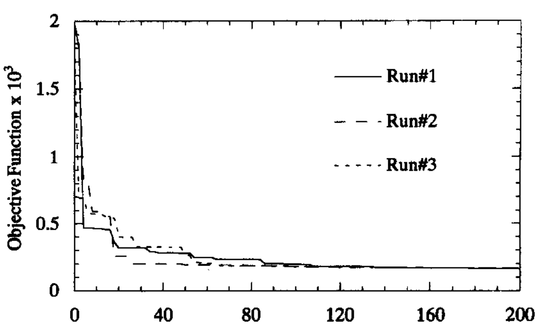

Fig. 1. Best generation results for simultaneous estimation of the single scattering albedo, the optical thickness and the phase function, with experimental errors $\sigma = 0.01$.

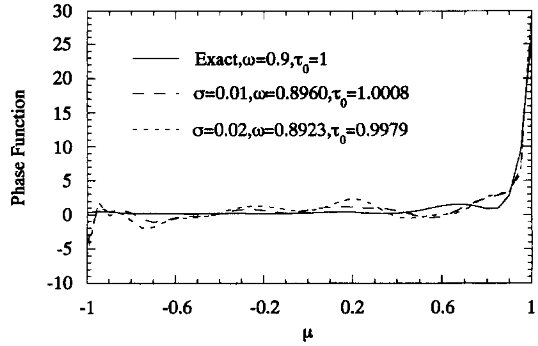

Fig. 2. Simultaneous estimation of the single scattering albedo, the optical thickness and the phase function, by inverse analysis with experimental errors $\sigma = 0.01$ and $\sigma = 0.02$.

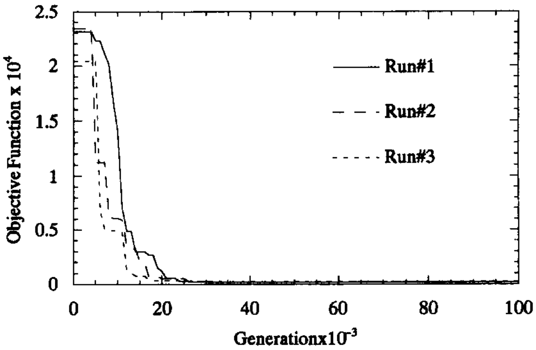

Fig. 3. Best generation results for simultaneous estimation of the single scattering albedo, the optical thickness and the phase function, with experimental errors $\sigma = 0.001$.

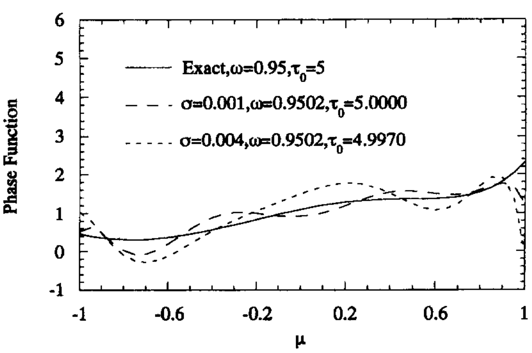

Fig. 4. Simultaneous estimation of the single scattering albedo, the optical thickness and the phase function, by inverse analysis with experimental errors $\sigma = 0.001$ and $\sigma = 0.004$.

analysis for a medium with single scattering albedo 0.95, optical thickness 5 and phase function II of Table 1 [21]. The best generation results in three different runs by the inverse analysis with experimental errors $\sigma = 0.001$ are shown in Fig. 3. Figure 4 shows the results obtained with the inverse analysis for simulated experimental data containing errors of standard deviation $\sigma = 0.001$ and $\sigma = 0.004$. The accuracy of the estimation for the single scattering albedo and the optical thickness is very good. The estimation of the phase function is more difficult than that of the single scattering albedo and the optical thickness. Comparing Figs. 1 and 2 with Figs. 3 and 4, we note that the accuracy of the estimation is more sensitive to the measurement errors as the optical thickness increased. For an optical thickness of 5, the measurement errors need to be much smaller than when the optical thickness is unity.

CONCLUSIONS

A genetic algorithm has been used to solve the inverse radiation problem for simultaneously determining the single scattering albedo, the optical thickness and the phase function, from the knowledge of the exit radiation intensities. The genetic algorithm is adopted as the optimizer to search the parameters of the radiation system according to the simulated radiation intensities exiting the system. The study shows that the single scattering albedo and the optical thickness can be estimated accurately even with noisy data. The estimation of the phase function is more difficult than that of the single scattering albedo and the optical thickness. As the optical thickness increased, the accuracy of the estimation becomes more sensitive to the measurement errors. The genetic algorithms are not efficient but are quite robust, i.e. the optimization procedure will yield a near-global optimal solution. It is suggested that the genetic algorithms are used when the traditional optimization methods fail, or are difficult to apply, such as in the inverse radiation problem considered here. It is evident that the genetic algorithms have the potential to be implemented in the field of inverse radiation problems.

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